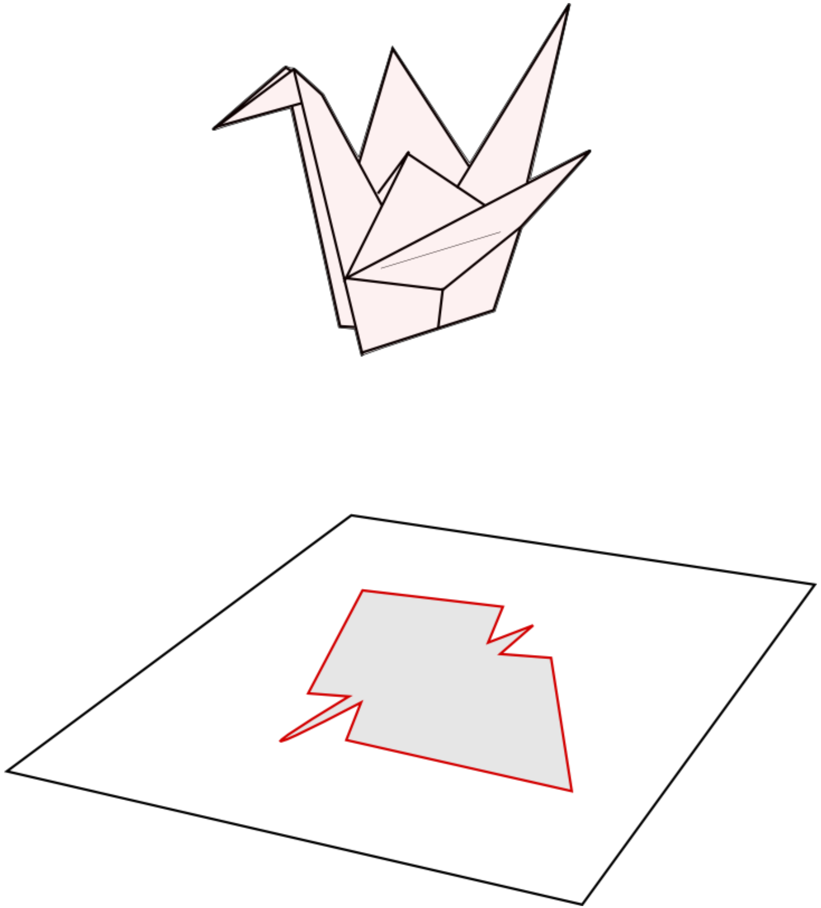
The Napkin Problem

Joe Slote & Sam Vinitsky

The Napkin Problem:

Definition: the maximum perimeter of an origami model is the largest perimeter you can measure by projecting your model onto the plane in any direction.



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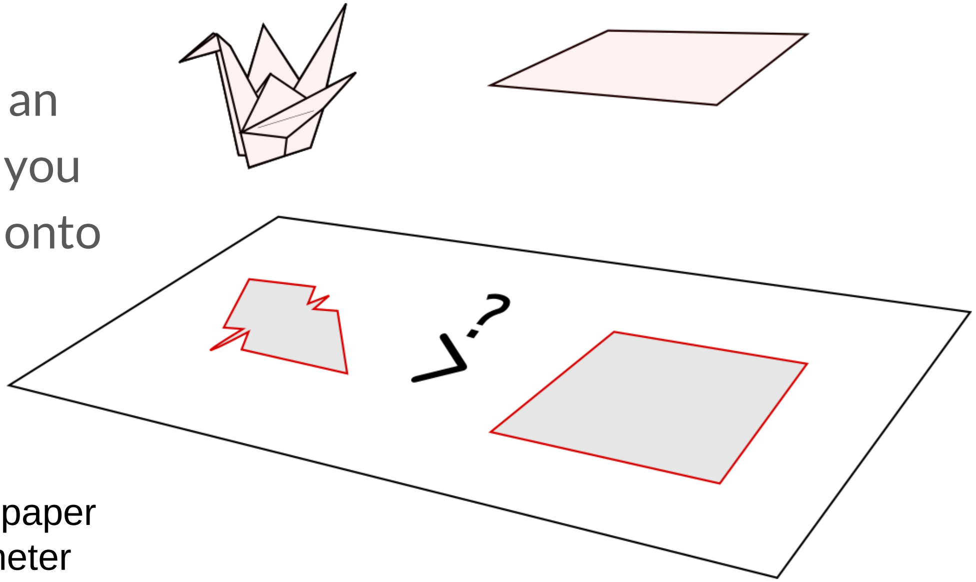

The Napkin Problem: Can you fold a square of paper into an origami model that has a maximum perimeter larger than the original square? If so, how much larger?

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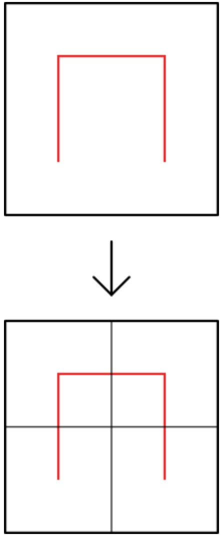
Answer:

Yes, and as large as we want!

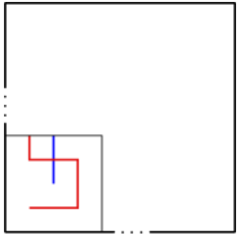
Key: — Previous Curve, — New Curve

Hilbert Curves: Recipe

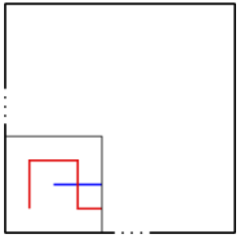
Step 1: Divide each grid square into fourths



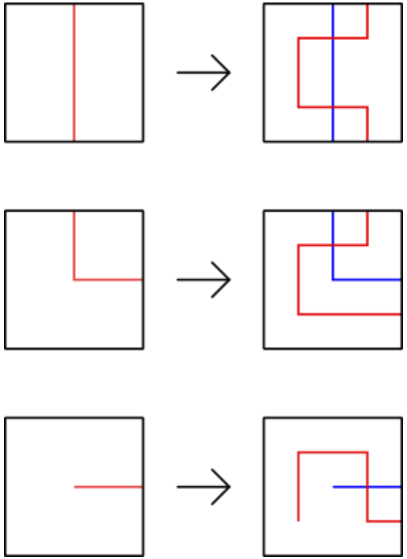
Step 2: Add the correct tile to the bottom-left.



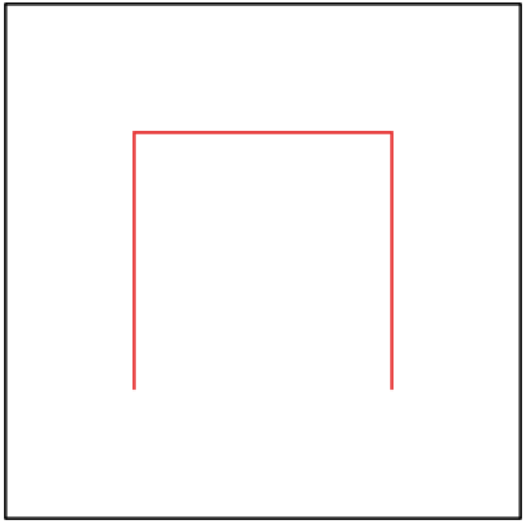
\- or -



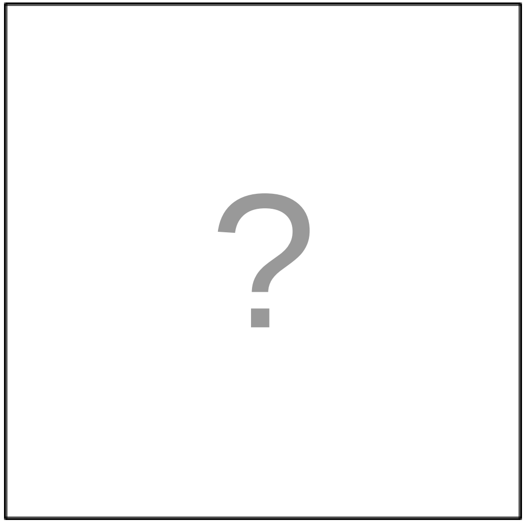
Step 3: Fill the rest of the grid with the correct tile



Hilbert Curves: Practice

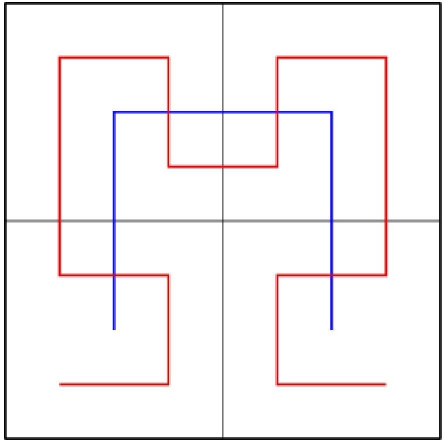


H_1

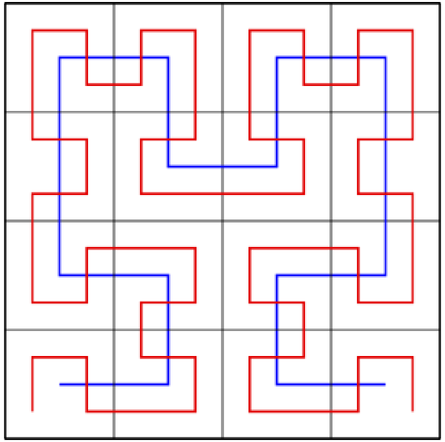


H_2

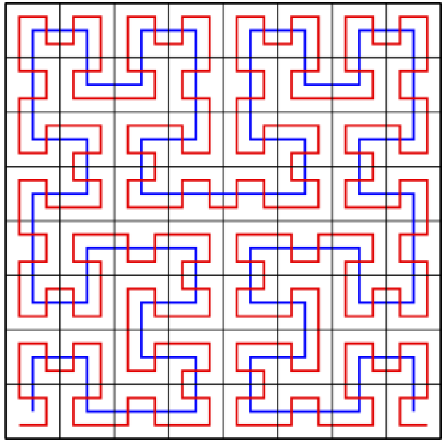
Hilbert Curves:



H_1 with H_2

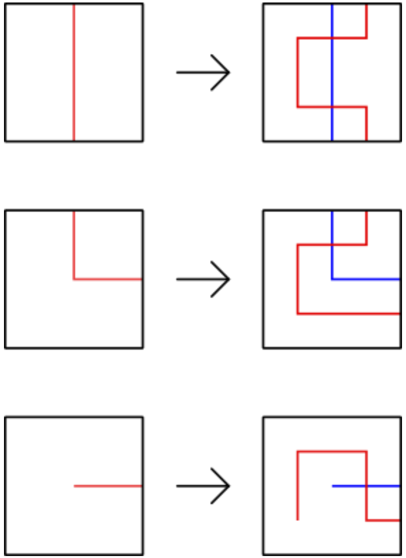


H_2 with H_3



H_3 with H_4

Hilbert Curves:



ℓ (old) = 1

ℓ (new) = 2

ℓ (old) = 1

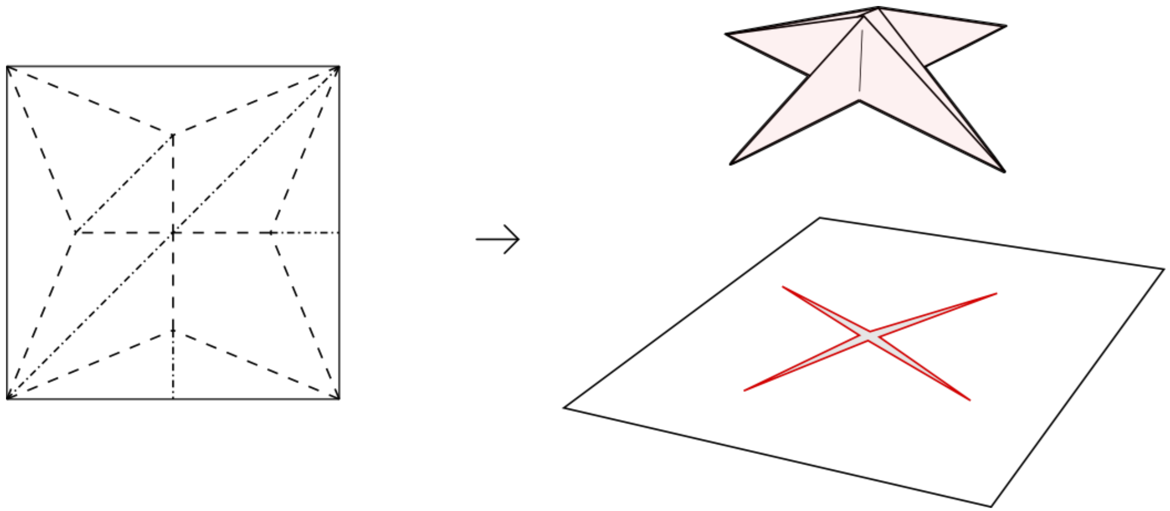
ℓ (new) = 2

ℓ (old) = 0.5

ℓ (new) = 1.75

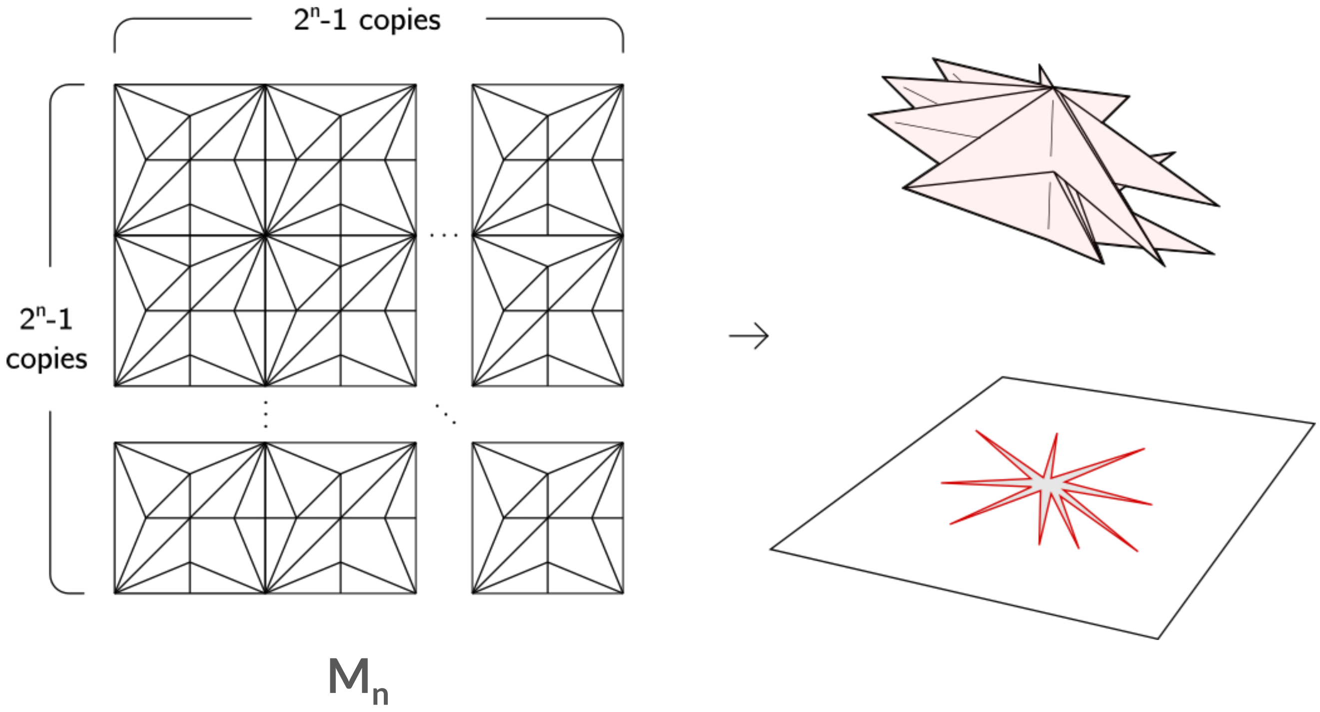
Conclusion: $\ell(H_{n+1}) \geq 2\ell(H_n)$

A Sequence of Origami Models



M_1

A Sequence of Origami Models

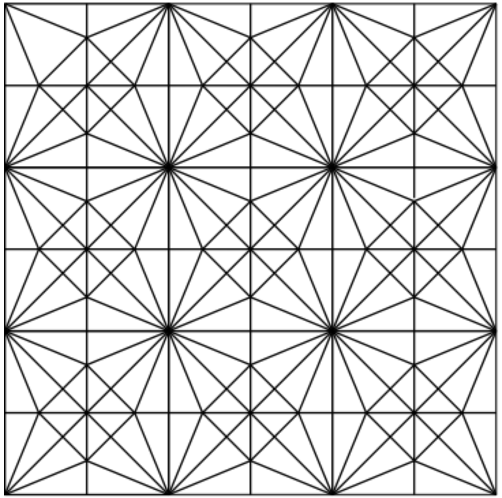


The Proof:

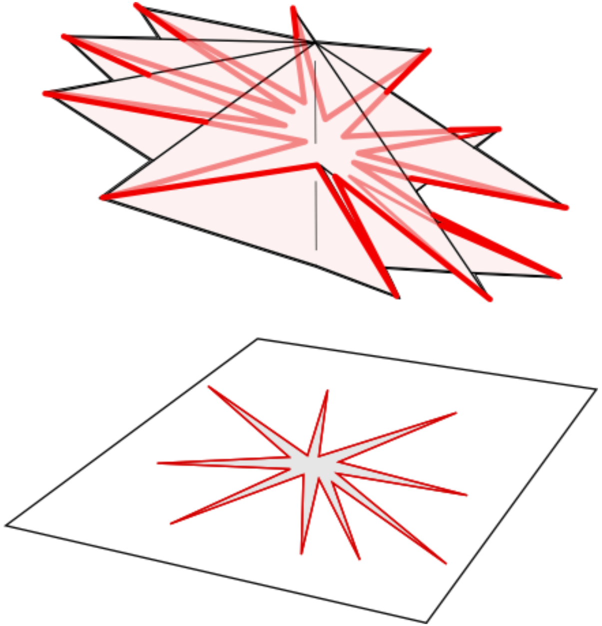
1. The length of the maximum perimeter of M_n is $\geq \ell(H_n)$
2. The sequence of $\ell(H_n)$s is unbounded, so neither is the sequence of maximum perimeters of M_n.

We need to show this.

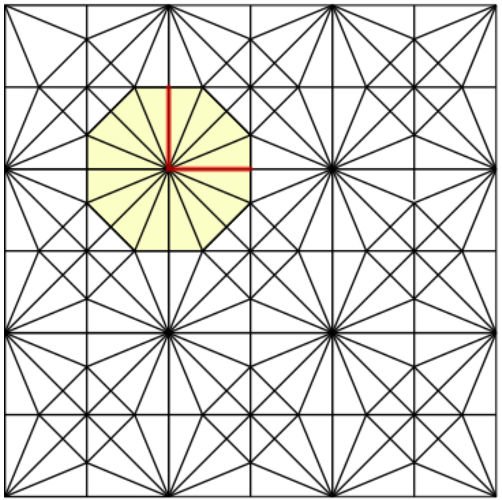
The Proof: Identifying the Perimeter



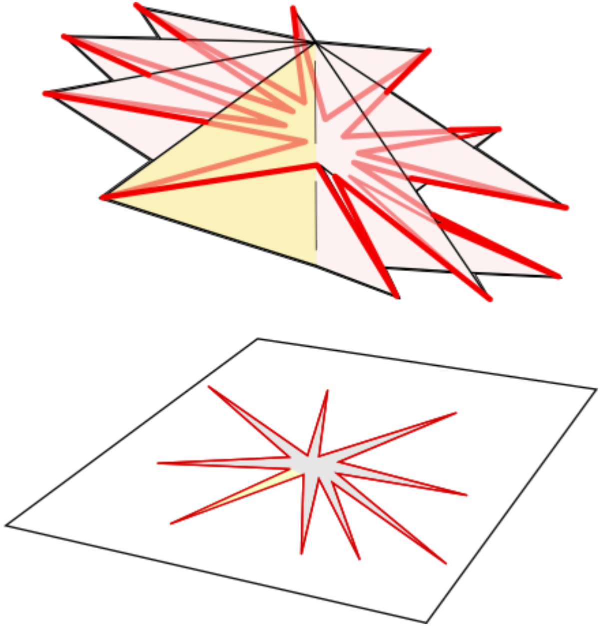
M_3



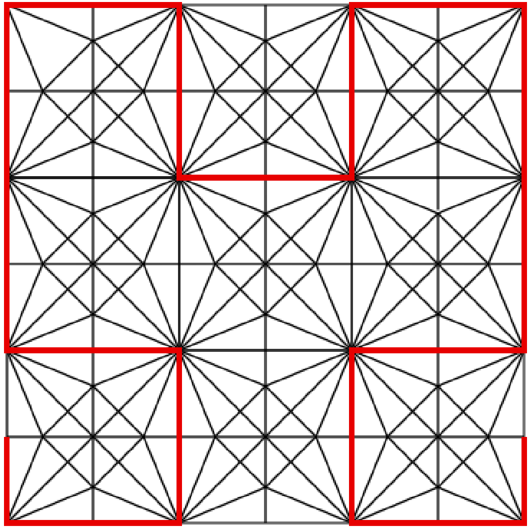
The Proof: Identifying the Perimeter



M_3



The Proof: Covering H_n



$\mathbf{M_3}$ covers $\mathbf{H_3}$